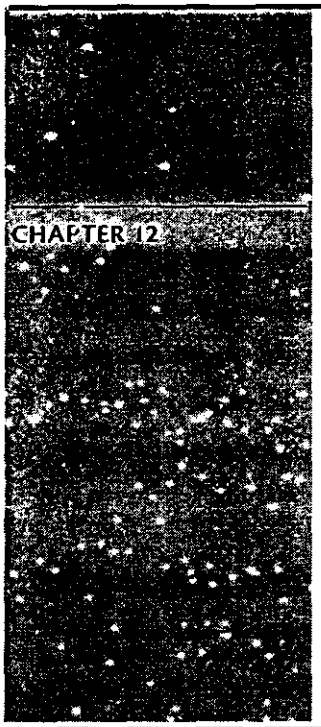

CHAPTER 12

Adiabatic Method

WE HAVE SEEN THAT THE POINT KINETICS method requires a complete solution of the space-time kinetics equations in order to determine the parameters, such as ρ, β_i, and Λ which play important roles in the system of differential equations governing this model. A first approach, the *point kinetics approximation*, supposes that the flux shape does not change during a transient. The initial flux of the non perturbed reactor is then used as a shape function. Unfortunately this hypothesis is wrong in many transients that reactors go through. The adiabatic method was one of the very first attempts to permit a variation of the shape function during a transient. It was probably invented from analogy with a very similar method in quantum physics. Before going into the mathematical derivation of the adiabatic method, we first analyze a situation in which point kinetics gives exact results.

Asymptotic Period

We consider the case of a nuclear reactor undergoing a very small perturbation relatively constant throughout the volume of the core. If the transient lasts for a sufficiently long time, then only the single less negative frequency will dominate transient behavior. All other contributions will be too small to contribute significantly. The reactor is the on an asymptotic period.

In this case, the fluxes of all energy groups, and the delayed neutron precursor concentrations all have the same exponential time dependence. This means that

$$\frac{\partial}{\partial t}C_i = \omega C_i$$

and

$$\frac{\partial}{\partial t}\phi_g = \omega \phi_g$$

or

$$\frac{\partial}{\partial t}[\phi] = \omega[\phi]$$

Consider now the space-time kinetics equations in matrix form, equation (20). The precursor equations become

$$\omega C_i = \beta_i [\nu \Sigma_f]^T [\phi] - \lambda_i C_i$$

If we isolate C_i from this equation to get

$$C_i = \frac{\beta[\nu\Sigma_f]^T[\phi]}{\omega + \lambda_i}$$

The flux equation becomes

$$[v]^{-1}\omega[\phi] = \nabla \cdot [D]\vec{\nabla}[\phi] - [\Sigma][\phi] + (1 - \beta)[\chi^p][\nu\Sigma_f]^T[\phi] + \sum_{i=1}^{D} [\chi_i^d]\lambda_i \frac{\beta_i[\nu\Sigma_f]^T[\phi]}{\omega + \lambda_i}$$

We add the terms $\sum_{i=1}^{D} [\chi_i^d]\beta_i[\nu\Sigma_f]^T[\phi]$ on each side to get

$$[v]^{-1}\omega[\phi] + \sum_{i=1}^{D} [\chi_i^d]\beta_i[\nu\Sigma_f]^T[\phi] - \sum_{i=1}^{D} [\chi_i^d]\lambda_i \frac{\beta_i[\nu\Sigma_f]^T[\phi]}{\omega + \lambda_i} = \nabla \cdot [D]\vec{\nabla}[\phi] - [\Sigma][\phi] + (1 - \beta)[\chi^p][\nu\Sigma_f]^T[\phi] + \sum_{i=1}^{D} [\chi_i^d]\beta_i[\nu\Sigma_f]^T[\phi]$$

or, if we regroup the terms in $[\nu\Sigma_f]$,

$$[v]^{-1}\omega[\phi] + \sum_{i=1}^{D}[\chi_i^d]\frac{\omega}{\omega+\lambda_i}\beta_i[\nu\Sigma_f]^T[\phi]$$

$$= \nabla\cdot[D]\vec{\nabla}[\phi] - [\Sigma][\phi] + (1-\beta)[\chi^p][\nu\Sigma_f]^T[\phi] \qquad \text{(EQ 90)}$$

$$+ \sum_{i=1}^{D}[\chi_i^d]\beta_i[\nu\Sigma_f]^T[\phi]$$

We now use the definition of reactivity (78) in which we use the flux as the shape function,

$$\rho = \frac{\langle[W]^T\left\{\nabla\cdot[D]\vec{\nabla}[\phi] - [\Sigma][\phi] + \left((1-\beta)[\chi^p] + \sum_{i=1}^{D}\beta[\chi_i^d]\right)\right\}[\nu\Sigma_f]^T[\phi]\rangle}{\langle[W]^T\left\{(1-\beta)[\chi^p] + \sum_{i=1}^{D}\beta[\chi_i^d]\right\}[\nu\Sigma_f]^T[\phi]\rangle}$$

which gives

$$\rho(t) = \frac{\langle[W]^T\left\{\omega[v]^{-1}[\phi] + \sum_{i=1}^{D}[\chi_i^d]\frac{\omega}{\omega+\lambda_i}\beta[\nu\Sigma_f]^T[\phi]\right\}\rangle}{\langle[W]^T\left\{(1-\beta)[\chi^p] + \sum_{i=1}^{D}\beta[\chi_i^d]\right\}[\nu\Sigma_f]^T[\phi]\rangle}$$

We now remove from the integrals the terms that do not depend on space. Also, we use the definitions of Λ and $\beta_i(t)$ to get

$$\rho(t) = \omega\Lambda(t) + \sum_{i=1}^{D} \frac{\omega}{\omega + \lambda_i}\beta_i$$

We thus have the result that the reactivity $\rho(t)$ is given by Nordheim's equation, which this time come directly from the rigorous space-time equations.

The motivation behind this demonstration is found in the left hand side of (90) which may be interpreted as a 1 over v absorber and a source term which would modify the fission source in a static equation.

Also, in an adiabatic transient, the flux shape does not vary in time, since all the fluxes and precursors vary at the same rate.

Adiabatic Approximation

In order to reduce the errors introduced by the point kinetics approximation, the shape function changes must be accounted for. We have seen in the previous section that in certain cases, the kinetics equations look like static equations.

The adiabatic method uses the idea to calculate the shape function as a solution of the static equations, with the instantaneous operators and parameters as they appear in the core during the transient.

We the use the definition $[\phi_t] \equiv [S(\vec{r}, t)]$

where $[\phi_t]$ is obtained by solving the static system

$$[L][\phi_t] = \frac{1}{\lambda_t}[M][\phi_t]$$

In other words, the shape function is obtained in calculating the flux in the perturbed reactor made artificially critical by adjusting the fission sources by the parameter λ_t.

In order to determine in which circumstances such an approximation could be justified, let us introduce the exponential transform of the flux and of the delayed neutron precursors,

$$\omega_g^p \equiv \frac{1}{\phi_g}\frac{\partial}{\partial t}\phi_g$$

and

$$\omega_i \equiv \frac{1}{C_i}\frac{\partial}{\partial t}C_i$$

Using these definitions, the precursor equations become

$$C_i(\vec{r}, t) = \frac{\beta_i}{\omega_i + \lambda_i}[\nu\Sigma_f]^T[\phi]$$

and the flux equations become after adding and substracting the term

$$\sum_{i=1}^{D} [\chi_i^d]\beta_i[\nu\Sigma_f]^T[\phi]$$

which gives

$$[v]^{-1}\omega[\phi] = \nabla \cdot [D]\vec{\nabla}[\phi] - [\Sigma][\phi] + \left((1-\beta)[\chi^p] + \sum_{i=1}^{D} [\chi_i^d]\beta_i\right)[\nu\Sigma_f]^T[\phi] - \sum_{i=1}^{D} [\chi_i^d]\frac{\omega_i\beta_i}{\lambda_i + \omega_i}[\nu\Sigma_f]^T[\phi]$$

We note that if the transient is slow, the ω_g that make the elements of the diagonal matrix $[\omega]^p$ will be small. Furthermore, after multiplication by the matrix $[v]^{-1}$ the left hand side will be even smaller, and very close to 0. If the ω_i do not depend on position, then this last equation looks much like the static equation, except that the delayed neutron source is adjusted to maintain reactor criticality. The usual static equation adjusts all neutron sources. If the delayed and prompt neutron spectra are alike, and if the delayed neutron source is relatively small, the flux shape will be very close to that obtained by static calculations. The adiabatic method will be quite good in this case.

Adiabatic Reactivity

The adiabatic approximation gives a very well known expression for the reactivity. We can write the definition of reactivity as,

$$\rho = \frac{\langle(-[L] + [M])[S]\rangle}{\langle[M][S]\rangle}$$

which becomes in adiabatic approximation,

$$\rho = \frac{\left\langle \left(-\frac{1}{\lambda_t}[M] + [M] \right)[S] \right\rangle}{\langle [M][S] \rangle}$$

in other words

$$\rho = \frac{\left\langle \left(1 - \frac{1}{\lambda_t} \right)[M][S] \right\rangle}{\langle [M][S] \rangle}$$

and thus

$$\rho = 1 - \frac{1}{\lambda_t}$$

Reactivity becomes

$$\rho = \frac{\lambda_t - 1}{\lambda_t}$$

In other words, adiabatic reactivity is obtained by taking the difference in the K_{eff} between two static calculations, one with the reactor in nominal state, the other in the perturbed state.

In fact, this is often used as the definition of reactivity. The restrictions imposed by the adiabatic method should be kept in mind when this definition is used.